Exam Symmetry in Physics

Date	April 10, 2015
Room	V 5161.0165
Time	14:00 - 17:00
Lecturer	D. Boer

- Write your name and student number on every separate sheet of paper
- You are not allowed to use the lecture notes, nor other notes or books
- All subquestions (a, b, etc) of the 3 exercises (18 in total) have equal weight
- Answers may be given in Dutch
- Illegible handwriting will be graded as incorrect
- Good luck!

Exercise 1

Consider an idealized beach ball with color pattern like in the picture below. If the north and south pole can be distinguished from each other, for instance by the color of the round patches, the symmetry group of the beach ball is a subgroup of $O(3)$ called C_{3v}.

(a) Identify all symmetry transformations that leave this beach ball invariant and divide them into conjugacy classes, using geometrical arguments.

(b) Give an identification between elements of C_{3v} and S_3 and argue that the two groups are isomorphic.

(c) Write down the character table of C_{3v} and explain how the entries are obtained.

(d) Construct explicitly the three-dimensional vector representation D^V for the two transformations that generate C_{3v} and check the determinants.

(e) Decompose D^V of C_{3v} into irreps and use this to conclude whether this group allows for an invariant three-dimensional vector.

(f) Determine the characters of the direct product representation $D^V \otimes D^V$ of C_{3v} and use them to determine the number of independent invariant tensors T^{ij} $(i, j = 1, 2, 3)$ (no need to construct them explicitly).

(g) Explain *in words* what changes if the north and south pole of the beach ball cannot be distinguished, i.e. explain what are the additional symmetries and whether an invariant three-dimensional vector is allowed or not.

Exercise 2

(a) Write down the defining representation of $SO(2)$.

(b) Write down the two-dimensional representation of $SO(2)$ obtained by its action on the vector

$$\begin{pmatrix} x+iy \\ x-iy \end{pmatrix}.$$

(c) Show whether the two above representations are equivalent or not.

(d) Explain whether the defining rep of $SO(2)$ is an irrep or not.

(e) Show that the following matrices do not form a representation of $SO(2)$:

$$D(\theta) = \begin{pmatrix} e^{i\theta/2} & 0 \\ 0 & e^{-i\theta/2} \end{pmatrix}, \tag{1}$$

for a rotation over an angle θ.

(f) Explain how wave functions transform under $SO(2)$ transformations.

Exercise 3

(a) Explain how a magnetic field transforms under rotations and reflections.

(b) Describe all symmetries of a three-dimensional space with a constant uniform magnetic field $\boldsymbol{B}$ pointing in the $\hat{z}$ direction (consider rotations, reflections, and translations).

(c) Explain which components of the (linear) momentum and the orbital angular momentum should be conserved based on the symmetries obtained in part (b).

Consider a particle with charge q moving with velocity $\boldsymbol{v}$ in this constant uniform magnetic field. The particle will experience a Lorentz force

$$\boldsymbol{F} = q\left(\boldsymbol{v} \times \boldsymbol{B}\right). \tag{2}$$

(d) Show that the left- and right-hand side of the Lorentz force equation (2) transform in the same way under rotations and reflections.

(e) Ignoring cyclotron radiation emitted by the particle, its trajectory will be circular. Explain whether this trajectory is in accordance with the symmetries in part (b) of this question, and in case not, explain why not.